

Paper Reference 9FM0/3A
Pearson Edexcel
Level 3 GCE

Further Mathematics

Advanced

PAPER 3A: Further Pure Mathematics 1

Wednesday 19 June 2024 – Afternoon

Time: 1 hour 30 minutes

YOU MUST HAVE

**Mathematical Formulae and Statistical Tables (Green),
calculator**

YOU WILL BE GIVEN

**Diagram Booklet
Answer Booklet**

X75686RA

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Booklet or on the separate diagrams – there may be more space than you need.

Do NOT write on this Question Paper.

You should show sufficient working to make your methods clear.

Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

**There are 10 questions in this Question Paper.
The total mark for this paper is 75.**

**The marks for EACH question are shown in brackets
– use this as a guide as to how much time to spend on
each question.**

**There may be spare copies of some diagrams in case
you need them.**

ADVICE

**Read each question carefully before you start to answer
it.**

Try to answer every question.

Check your answers if you have time at the end.

1. (a) Refer to the table for Question 1 in the Diagram Booklet.

Given that

$$y = \ln(3 + x^2)$$

complete the table in the Diagram Booklet with the value of y corresponding to $x = 3$, giving your answer to 4 significant figures.

(1 mark)

(continued on the next page)

1. continued.

In part (b) you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

- (b) Use Simpson's rule with all the values of y in the completed table to estimate, to 3 significant figures, the value of

$$\int_2^5 \ln(3 + x^2) dx$$

(3 marks)

- (c) Using your answer to part (b) and making your method clear, estimate the value of

$$\int_2^5 \ln \sqrt{(3 + x^2)} dx$$

(1 mark)

(Total for Question 1 is 5 marks)

2. Use algebra to determine the values of x for which

$$|x^2 - 2x| \leq x$$

(Total for Question 2 is 4 marks)

3. Use L'Hospital's rule to show that

$$\lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right) = 0$$

(Total for Question 3 is 6 marks)

4. Refer to the information for Question 4 in the Diagram Booklet.

It shows the Taylor series expansion of $f(x)$ about $x = a$

The curve with equation

$y = f(x)$ satisfies the differential equation

$$\cos x \frac{d^2 y}{dx^2} + y^2 \frac{dy}{dx} + \sin x = 0$$

Given that $\left(\frac{\pi}{4}, 1\right)$ is a stationary point of the curve,

- (a) determine the nature of this stationary point, giving a reason for your answer.
(2 marks)

- (b) Show that

$$\frac{d^3 y}{dx^3} = \sqrt{2} - 2 \text{ at this stationary point.}$$

(4 marks)

(continued on the next page)

Turn over

4. continued.

(c) Hence determine a series solution for y , in ascending powers of

$\left(x - \frac{\pi}{4}\right)$ up to and including the term in

$\left(x - \frac{\pi}{4}\right)^3$, giving each coefficient in simplest form.

(2 marks)

(Total for Question 4 is 8 marks)

5. $y = e^{3x} \sin x$

(a) Use Leibnitz's theorem to show that

$$\frac{d^4 y}{dx^4} = 28e^{3x} \sin x + 96e^{3x} \cos x$$

(6 marks)

(b) Hence express $\frac{d^4 y}{dx^4}$ in the form

$$Re^{3x} \sin(x + \alpha)$$

where R and α are constants to be determined,

$$R > 0 \text{ and } 0 < \alpha < \frac{\pi}{2}$$

(3 marks)

(Total for Question 5 is 9 marks)

6. The ellipse **E** has equation

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

The hyperbola **H** has equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

where **a** and **b** are positive constants.

Given that

- the eccentricity of **H** is the reciprocal of the eccentricity of **E**
- the coordinates of the foci of **H** are the same as the coordinates of the foci of **E**

determine

(i) the value of **a**

(ii) the value of **b**

(Total for Question 6 is 6 marks)

7. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

- (a) Use the substitution

$t = \tan\left(\frac{\theta}{2}\right)$ to show that

$$\int \frac{1}{2\sin\theta + \cos\theta + 2} d\theta = \int \frac{a}{(t+b)^2 + c} dt$$

where a , b and c are constants to be determined.

(3 marks)

(continued on the next page)

7. continued.

(b) Hence show that

$$\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} \frac{1}{2\sin\theta + \cos\theta + 2} d\theta = \ln\left(\frac{2\sqrt{3}}{3}\right)$$

(4 marks)

(Total for Question 7 is 7 marks)

8. The parabola **P** has equation $y^2 = 4ax$, where **a** is a positive constant.

The point

A($at^2, 2at$), where
 $t \neq 0$, lies on **P**

- (a) Use calculus to show that an equation of the tangent to **P** at **A** is

$$yt = x + at^2$$

(3 marks)

(continued on the next page)

8. continued.

The point

$B(2k^2, 4k)$ and the point

$C(2k^2, -4k)$, where k is a constant, lie on P

The tangent to P at B and the tangent to P at C
intersect at the point D

Given that the area of the triangle BCD is 432

- (b) determine the coordinates of B and the
coordinates of C
(5 marks)

(Total for Question 8 is 8 marks)

9. (i) The line L_1 has equation

$$\underline{r} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ -1 \end{pmatrix}$$

The line L_2 has equation

$$\underline{r} = \begin{pmatrix} 13 \\ 5 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$$

where λ and μ are scalar parameters.

The lines L_1 and L_2 intersect at the point P

- (a) Determine the coordinates of P
(2 marks)

(continued on the next page)

9. (i) continued.

Given that the plane Π contains both L_1 and L_2

(b) determine a Cartesian equation for Π
(4 marks)

(ii) Determine a Cartesian equation for each of the two lines that

- pass through $(0, 0, 0)$
- make an angle of 60° with the x -axis
- make an angle of 45° with the y -axis

(4 marks)

(Total for Question 9 is 10 marks)

10. The motion of a particle **P** along the **x**-axis is modelled by the differential equation

$$t^2 \frac{d^2x}{dt^2} - 2t(t+1) \frac{dx}{dt} + 2(t+1)x = 8t^3 e^t \quad (\text{I})$$

where **P** has displacement **x** metres from the origin **O** at time **t** minutes, $t > 0$

- (a) Show that the transformation $x = tu$ transforms the differential equation (I) into the differential equation

$$\frac{d^2u}{dt^2} - 2 \frac{du}{dt} = 8e^t$$

(4 marks)

(continued on the next page)

10. continued.

Given that **P** is at **O** when

$t = \ln 3$ and when

$t = \ln 5$

- (b) determine the particular solution of the
differential equation (I)
(8 marks)

(Total for Question 10 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER
